The Dwarf Signal

A Railway Signalling Device

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Overview

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Dwarf Railway Signals

- Dwarf signals serve the same purpose as typical high signals.

- They are small, short, wayside or trackside signals.

- Commonly found where there is restricted room between the tracks.

- Short enough to fit notch at bottom of normal clearance envelope.

- Low position limits distant visibility: only relatively slow-speed track.

- High-speed, poor clearance: signal bridge is more expensive solution.

- Frequently used for:

  - Transition tracks: sidings, spurs, branch lines, turn-outs, diamonds.

  - Signalling limited clearances, reduced speeds.

  - Alternative to standard high signals to reduce costs and maintenance.

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Our 3-Lamp Dwarf Signal

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Different Aspects of the Dwarf Signal

- Our dwarf signal consists of three white lamps.

- Lamps can show two aspects: on (burning) 🟡 and off (not burning) ⚪.

- If all equipment is physically intact, there are four different valid aspects:

Lamps   Dark   Stop

Warning   Drive

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Interpreting the Different Aspects

- The safest aspect is stop.

- The dark aspect is used only to prolong the lifetime of the lamps.

- The dark aspect should never be seen by a driver in normal conditions.

- If drive can't be shown, warning should be shown.

- If warning can't be shown, stop should be shown instead.

- Drivers are assumed to interpret an invalid aspect in a safe way:

  - dark is interpreted as stop.

  - L1 burning on its own is interpreted as stop.

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Rules for Switching between Aspects

- Only one lamp aspect may be changed at one time.

- The three lamps must never be on simultaneously.

- The signal must never be dark unless:

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- The change to and from dark is allowed only to and from stop.

- Initial aspect and aspect in powerless state of actors is stop.

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Overview

Formal Specification of the Dwarf Signal

- There are three lamp identifiers: $LampId ::= L1 \mid L2 \mid L3$.

- There are four aspects of the signal:

$$dark, stop, warning, drive : \mathbb{F}\ LampId$$

$$dark = \emptyset$$
$$stop = \{L1, L2\}$$
$$warning = \{L1, L3\}$$
$$drive = \{L2, L3\}$$

- These are proper aspects of the signal.

- Transient aspects: $ProperState == \{dark, stop, warning, drive\}$.

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State Model for Dwarf Signal

We model the dwarf signal using six state variables:

- *last_proper_state*: constrains signal transitions between proper aspects.

- *turn_off*: lamps that must be extinguished to reach desired state.

- *turn_on*: lamps that must be lit to reach desired state.

- *last_state*: constrains sequence of signal states (proper or transient).

- *required lamps*: lamps to turn on/off at a time.

- *current_state*: set of lamps identifiers currently burning.

- *desired_proper_state*: the next desired proper aspect of the signal.

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State Invariants

Two invariants link the state components:

Invariant 1 The changes indicated by the two sets $turn\_off$ and $turn\_on$ will transform the current state into the desired state.

Invariant 2 It can never be the case that a lamp needs to be both turned on and turned off to reach the desired proper state.

A consequence of these two invariants:

Desired state reached when there are no more lamps to turn off or on.

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last_proper_state : ProperState
turn_off , turn_on : $\mathbb{F}$ LampId
last_state , current_state : Signal
desired_proper_state : ProperState
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(current_state \ turn_off) ∪ turn_on = desired_proper_state
turn_off ∩ turn_on = ∅

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___ *Dwarf* _____
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$turn\_off, turn\_on : \mathbb{F}\ LampId$
$last\_state, current\_state : Signal$
$desired\_proper\_state : ProperState$
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$(current\_state \setminus turn\_off) \cup turn\_on = desired\_proper\_state$
$turn\_off \cap turn\_on = \emptyset$

Formal Specification of the State of the Dwarf Signal

Dwarf _____

$last\_proper\_state$: $ProperState$
$turn\_off$, $turn\_on$: $\mathbb{F}\ LampId$
$last\_state$, $current\_state$: $Signal$
$desired\_proper\_state$: $ProperState$

_____

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│ *last_state*, *current_state* : *Signal*
│ *desired_proper_state* : *ProperState*
├───────────────────
│ (*current_state* \ *turn_off*) ∪ *turn_on* = *desired_proper_state*
│ *turn_off* ∩ *turn_on* = ∅
└─────────────────────────────────

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$last\_proper\_state$: *ProperState*
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$$
\begin{array}{l}
\rule{5cm}{0.4pt} \; Dwarf \; \rule{5cm}{0.4pt} \\
last\_proper\_state : ProperState \\
turn\_off, turn\_on : \mathbb{F}\ LampId \\
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```
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│ last_proper_state : ProperState
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Overview

Formalising the Safety Requirements

- There are five safety requirements for the dwarf signal.

- We specify each safety requirement separately.

- This principle is known as separation of concerns.

- Each safety requirement is a constraint on the dwarf signal state.

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Safety 1: The signal should never light all its lamps

NeverShowAll
Dwarf

$current\_state \neq \{L1, L2, L3\}$

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$NeverShowAll$
$Dwarf$

$current\_state \neq \{L1, L2, L3\}$

Safety 2: Only one lamp should be changed at a time

MaxOneLampChange
Dwarf

$\exists\, l : LampId \bullet$
    $current\_state \setminus last\_state = \{l\}$
    $\vee$
    $last\_state \setminus current\_state = \{l\}$
    $\vee$
    $last\_state = current\_state$

Safety 2: Only one lamp should be changed at a time

$\underline{\quad MaxOneLampChange \quad}$
$Dwarf$

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```
┌─ MaxOneLampChange ─────────────────────────────────
│ Dwarf
├─────────────────────────────────────────────────────
│ ∃ l : LampId •
│     current_state \ last_state = {l}
│     ∨
│     last_state \ current_state = {l}
│     ∨
│     last_state = current_state
└─────────────────────────────────────────────────────
```

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├─────────────────────────────────────────────────
│ ∃ l : LampId •
│     current_state \ last_state = {l}
│     ∨
│     last_state \ current_state = {l}
│     ∨
│     last_state = current_state
└─────────────────────────────────────────────────
```

Safety 3: It is forbidden to change signal from stop to drive

$ForbidStopToDrive$
$Dwarf$

$last\_proper\_state = stop \Rightarrow desired\_proper\_state \neq drive$

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$ForbidStopToDrive$
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$last\_proper\_state = stop \Rightarrow desired\_proper\_state \neq drive$

Safety 4: The only proper aspect following dark is stop

DarkOnlyToStop
Dwarf

$last\_proper\_state = dark \Rightarrow desired\_proper\_state = stop$

Safety 4: The only proper aspect following dark is stop

DarkOnlyToStop
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$last\_proper\_state = dark \Rightarrow desired\_proper\_state = stop$

DarkOnlyFromStop
Dwarf

$desired\_proper\_state = dark \Rightarrow last\_proper\_state = stop$

Safety 5: The only proper aspect preceding dark is stop

DarkOnlyFromStop
Dwarf

$desired\_proper\_state = dark \Rightarrow last\_proper\_state = stop$

Safety 5: The only proper aspect preceding dark is stop

$DarkOnlyFromStop$
$Dwarf$

$desired\_proper\_state = dark \Rightarrow last\_proper\_state = stop$

Safety 5: The only proper aspect preceding dark is stop

DarkOnlyFromStop ──────────────────────────────
Dwarf
──────────
$desired\_proper\_state = dark \Rightarrow last\_proper\_state = stop$

Safety Specification for the Dwarf Signal

- Each safety requirement is a constraint on the state *Dwarf*.

- The dwarf signal must satisfy all five safety requirements.

- The *DwarfSignal* state is the safe restriction of the *Dwarf*.

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$DwarfSignal$
$NeverShowAll$
$MaxOneLampChange$
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\begin{array}{l}
\underline{DwarfSignal} \\
NeverShowAll \\
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ForbidStopToDrive \\
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DarkOnlyFromStop \\
\end{array}
$$

Overview

Initialisation for the Dwarf Signal

Initially, the dwarf signal shows the stop aspect:

Init
_____
DwarfSignal'
_____

$last\_proper\_state' = stop$
$turn\_off' = \emptyset$
$turn\_on' = \emptyset$
$last\_state' = stop$
$current\_state' = stop$
$desired\_proper\_state' = stop$

Initialisation for the Dwarf Signal

Initially, the dwarf signal shows the stop aspect:

$$
\begin{array}{l}
\textit{Init} \underline{\hspace{6cm}} \\
\textit{DwarfSignal}' \\
\underline{\hspace{3cm}} \\
\textit{last\_proper\_state}' = \textit{stop} \\
\textit{turn\_off}' = \emptyset \\
\textit{turn\_on}' = \emptyset \\
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\textit{last\_proper\_state}' = \textit{stop}\\
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\end{array}
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Initialisation for the Dwarf Signal

Initially, the dwarf signal shows the stop aspect:

$\begin{array}{l}
\hline
\textit{Init} \underline{\hspace{8cm}} \\
\quad DwarfSignal' \\
\hline
\quad last_proper_state' = stop \\
\quad turn_off' = \emptyset \\
\quad turn_on' = \emptyset \\
\quad last_state' = stop \\
\quad current_state' = stop \\
\quad desired_proper_state' = stop \\
\hline
\end{array}$

Initialisation for the Dwarf Signal

Initially, the dwarf signal shows the stop aspect:

$\underline{\quad Init \rule{8cm}{0.4pt}}$
$DwarfSignal'$
$\overline{\rule{3cm}{0.4pt}}$
$last\_proper\_state' = stop$
$turn\_off' = \emptyset$
$turn\_on' = \emptyset$
$last\_state' = stop$
$current\_state' = stop$
$desired\_proper\_state' = stop$

Operations on the Dwarf Signal

There are three operations to change the state of the dwarf signal:

SetNewProperState The operator (the railway signaller) sets the new desired proper state. This is the only external operation.

TurnOff An internal operation that turns off one of the lamps needed to make progress towards the new desired state.

TurnOn Another internal operation that turns on one of the lamps needed to make progress towards the new desired state.

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Setting the New Desired Proper State

- Operation on the state *DwarfSignal* with one input: $st?$: *ProperState*.
- Preconditions: no outstanding actions and a genuinely new proper state.

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- Preconditions: no outstanding actions and a genuinely new proper state.

```
┌─ SetNewProperState ──────────────────────────────
│ ΔDwarfSignal
│ st? : ProperState
│ ──────────────────────────────────────────
│ current_state = desired_proper_state
│ st? ≠ current_state
│ last_proper_state' = current_state
│ turn_off' = current_state \ st?
│ turn_on' = st? \ current_state
│ last_state' = current_state
│ current_state' = current_state
│ desired_proper_state' = st?
└──────────────────────────────────────────────────
```

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- Operation on the state $DwarfSignal$ with one input: $st? : ProperState$.
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$\begin{array}{l}
\underline{\quad SetNewProperState \quad} \\
\quad \Delta DwarfSignal \\
\quad st? : ProperState \\
\underline{} \\
\quad current_state = desired_proper_state \\
\quad st? \neq current_state \\
\quad last_proper_state' = current_state \\
\quad turn_off' = current_state \setminus st? \\
\quad turn_on' = st? \setminus current_state \\
\quad last_state' = current_state \\
\quad current_state' = current_state \\
\quad desired_proper_state' = st?
\end{array}$

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\begin{array}{l}
\underline{\quad SetNewProperState \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad} \\
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current\_state = desired\_proper\_state \\
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current\_state' = current\_state \\
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\end{array}
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st? \neq current\_state \\
last\_proper\_state' = current\_state \\
turn\_off' = current\_state \setminus st? \\
turn\_on' = st? \setminus current\_state \\
last\_state' = current\_state \\
current\_state' = current\_state \\
desired\_proper\_state' = st?
\end{array}
$$

Setting the New Desired Proper State

- Operation on the state $DwarfSignal$ with one input: $st? : ProperState$.
- Preconditions: no outstanding actions and a genuinely new proper state.

$$
\begin{array}{l}
\underline{SetNewProperState} \\
\Delta DwarfSignal \\
st? : ProperState \\
\hline
current\_state = desired\_proper\_state \\
st? \neq current\_state \\
last\_proper\_state' = current\_state \\
turn\_off' = current\_state \setminus st? \\
turn\_on' = st? \setminus current\_state \\
last\_state' = current\_state \\
current\_state' = current\_state \\
desired\_proper\_state' = st?
\end{array}
$$

Setting the New Desired Proper State

- Operation on the state $DwarfSignal$ with one input: $st? : ProperState$.
- Preconditions: no outstanding actions and a genuinely new proper state.

$$
\begin{array}{l}
\underline{\quad SetNewProperState \quad} \\
\Delta DwarfSignal \\
st? : ProperState \\
\hline
current\_state = desired\_proper\_state \\
st? \neq current\_state \\
last\_proper\_state' = current\_state \\
turn\_off' = current\_state \setminus st? \\
turn\_on' = st? \setminus current\_state \\
last\_state' = current\_state \\
current\_state' = current\_state \\
desired\_proper\_state' = st?
\end{array}
$$

Setting the New Desired Proper State

- Operation on the state $DwarfSignal$ with one input: $st? : ProperState$.
- Preconditions: no outstanding actions and a genuinely new proper state.

$SetNewProperState$
$\Delta DwarfSignal$
$st? : ProperState$

$current\_state = desired\_proper\_state$
$st? \neq current\_state$
$last\_proper\_state' = current\_state$
$turn\_off' = current\_state \setminus st?$
$turn\_on' = st? \setminus current\_state$
$last\_state' = current\_state$
$current\_state' = current\_state$
$desired\_proper\_state' = st?$

Setting the New Desired Proper State

- Operation on the state $DwarfSignal$ with one input: $st? : ProperState$.
- Preconditions: no outstanding actions and a genuinely new proper state.

$\quad$ *SetNewProperState*
$\quad$ $\Delta DwarfSignal$
$\quad$ $st? : ProperState$

$\quad$ $current\_state = desired\_proper\_state$
$\quad$ $st? \neq current\_state$
$\quad$ $last\_proper\_state' = current\_state$
$\quad$ $turn\_off' = current\_state \setminus st?$
$\quad$ $turn\_on' = st? \setminus current\_state$
$\quad$ $last\_state' = current\_state$
$\quad$ $current\_state' = current\_state$
$\quad$ $desired\_proper\_state' = st?$

Setting the New Desired Proper State

- Operation on the state $DwarfSignal$ with one input: $st? : ProperState$.
- Preconditions: no outstanding actions and a genuinely new proper state.

$$
\begin{array}{l}
\underline{\quad SetNewProperState \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad} \\
\Delta DwarfSignal \\
st? : ProperState \\
\underline{\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad} \\
current\_state = desired\_proper\_state \\
st? \neq current\_state \\
last\_proper\_state' = current\_state \\
turn\_off' = current\_state \setminus st? \\
turn\_on' = st? \setminus current\_state \\
last\_state' = current\_state \\
current\_state' = current\_state \\
desired\_proper\_state' = st?
\end{array}
$$

Turning off a Lamp

- Operation on the state *DwarfSignal* with one input: $l? : LampId$.
- Precondition: we must have a requirement to turn $l?$ off.

Turning off a Lamp

- Operation on the state $DwarfSignal$ with one input: $l? : LampId$.
- Precondition: we must have a requirement to turn $l?$ off.

Turning off a Lamp

- Operation on the state $DwarfSignal$ with one input: $l? : LampId$.
- Precondition: we must have a requirement to turn $l?$ off.

$$
\begin{array}{l}
\textit{TurnOff} \\
\hline
\Delta \textit{DwarfSignal} \\
l? : \textit{LampId} \\
\hline
l? \in \textit{turn\_off} \\
\textit{last\_proper\_state}' = \textit{last\_proper\_state} \\
\textit{turn\_off}' = \textit{turn\_off} \setminus \{l?\} \\
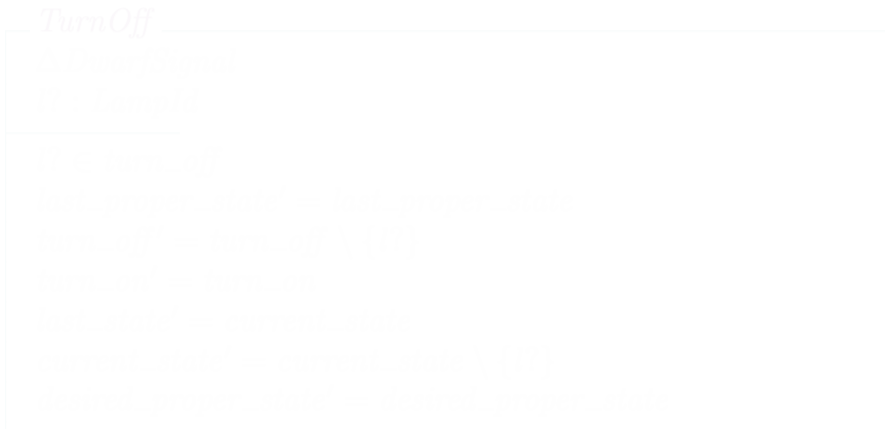\textit{turn\_on}' = \textit{turn\_on} \\
\textit{last\_state}' = \textit{current\_state} \\
\textit{current\_state}' = \textit{current\_state} \setminus \{l?\} \\
\textit{desired\_proper\_state}' = \textit{desired\_proper\_state}
\end{array}
$$

Turning off a Lamp

- Operation on the state $DwarfSignal$ with one input: $l? : LampId$.
- Precondition: we must have a requirement to turn $l?$ off.

$$
\begin{array}{l}
\underline{\quad TurnOff \quad\rule{8cm}{0pt}} \\
\Delta DwarfSignal \\
l? : LampId \\
\underline{\rule{4cm}{0pt}} \\
l? \in turn\_off \\
last\_proper\_state' = last\_proper\_state \\
turn\_off' = turn\_off \setminus \{l?\} \\
turn\_on' = turn\_on \\
last\_state' = current\_state \\
current\_state' = current\_state \setminus \{l?\} \\
desired\_proper\_state' = desired\_proper\_state
\end{array}
$$

Turning off a Lamp

- Operation on the state $DwarfSignal$ with one input: $l? : LampId$.
- Precondition: we must have a requirement to turn $l?$ off.

$$
\begin{array}{l}
\underline{TurnOff} \\
\Delta DwarfSignal \\
l? : LampId \\
\hline
l? \in turn\_off \\
last\_proper\_state' = last\_proper\_state \\
turn\_off' = turn\_off \setminus \{l?\} \\
turn\_on' = turn\_on \\
last\_state' = current\_state \\
current\_state' = current\_state \setminus \{l?\} \\
desired\_proper\_state' = desired\_proper\_state
\end{array}
$$

Turning off a Lamp

- Operation on the state $DwarfSignal$ with one input: $l? : LampId$.
- Precondition: we must have a requirement to turn $l?$ off.

$TurnOff$
$\Delta DwarfSignal$
$l? : LampId$

$l? \in turn\_off$
$last\_proper\_state' = last\_proper\_state$
$turn\_off' = turn\_off \setminus \{l?\}$
$turn\_on' = turn\_on$
$last\_state' = current\_state$
$current\_state' = current\_state \setminus \{l?\}$
$desired\_proper\_state' = desired\_proper\_state$

Turning off a Lamp

- Operation on the state $DwarfSignal$ with one input: $l? : LampId$.
- Precondition: we must have a requirement to turn $l?$ off.

$TurnOff$ _____
$\Delta DwarfSignal$
$l? : LampId$

_____

$l? \in turn\_off$
$last\_proper\_state' = last\_proper\_state$
$turn\_off' = turn\_off \setminus \{l?\}$
$turn\_on' = turn\_on$
$last\_state' = current\_state$
$current\_state' = current\_state \setminus \{l?\}$
$desired\_proper\_state' = desired\_proper\_state$

Turning off a Lamp

- Operation on the state $DwarfSignal$ with one input: $l? : LampId$.
- Precondition: we must have a requirement to turn $l?$ off.

$$
\begin{array}{l}
\underline{\quad TurnOff \quad\rule{6cm}{0pt}} \\
\Delta DwarfSignal \\
l? : LampId \\
\underline{\rule{3cm}{0pt}} \\
l? \in turn\_off \\
last\_proper\_state' = last\_proper\_state \\
turn\_off' = turn\_off \setminus \{l?\} \\
turn\_on' = turn\_on \\
last\_state' = current\_state \\
current\_state' = current\_state \setminus \{l?\} \\
desired\_proper\_state' = desired\_proper\_state
\end{array}
$$

Turning off a Lamp

- Operation on the state $DwarfSignal$ with one input: $l? : LampId$.
- Precondition: we must have a requirement to turn $l?$ off.

__ $TurnOff$ _____
$\Delta DwarfSignal$
$l? : LampId$

$l? \in turn\_off$
$last\_proper\_state' = last\_proper\_state$
$turn\_off' = turn\_off \setminus \{l?\}$
$turn\_on' = turn\_on$
$last\_state' = current\_state$
$current\_state' = current\_state \setminus \{l?\}$
$desired\_proper\_state' = desired\_proper\_state$

Turning off a Lamp

- Operation on the state $DwarfSignal$ with one input: $l? : LampId$.
- Precondition: we must have a requirement to turn $l?$ off.

$$
\begin{array}{l}
\underline{\ TurnOff\ } \\
\Delta DwarfSignal \\
l? : LampId \\
\hline
l? \in turn\_off \\
last\_proper\_state' = last\_proper\_state \\
turn\_off' = turn\_off \setminus \{l?\} \\
turn\_on' = turn\_on \\
last\_state' = current\_state \\
current\_state' = current\_state \setminus \{l?\} \\
desired\_proper\_state' = desired\_proper\_state
\end{array}
$$

Turning off a Lamp

- Operation on the state $DwarfSignal$ with one input: $l? : LampId$.
- Precondition: we must have a requirement to turn $l?$ off.

$$
\begin{array}{l}
\underline{\quad TurnOff \quad\rule{4cm}{0pt}} \\
\Delta DwarfSignal \\
l? : LampId \\
\hline
l? \in turn\_off \\
last\_proper\_state' = last\_proper\_state \\
turn\_off' = turn\_off \setminus \{l?\} \\
turn\_on' = turn\_on \\
last\_state' = current\_state \\
current\_state' = current\_state \setminus \{l?\} \\
desired\_proper\_state' = desired\_proper\_state
\end{array}
$$

Turning off a Lamp

- Operation on the state $DwarfSignal$ with one input: $l? : LampId$.
- Precondition: we must have a requirement to turn $l?$ off.

$$
\begin{array}{l}
\underline{\quad TurnOff \quad\rule{4cm}{0.4pt}} \\
\Delta DwarfSignal \\
l? : LampId \\
\hline
l? \in turn\_off \\
last\_proper\_state' = last\_proper\_state \\
turn\_off' = turn\_off \setminus \{l?\} \\
turn\_on' = turn\_on \\
last\_state' = current\_state \\
current\_state' = current\_state \setminus \{l?\} \\
desired\_proper\_state' = desired\_proper\_state
\end{array}
$$

Turning off a Lamp

- Operation on the state $DwarfSignal$ with one input: $l? : LampId$.
- Precondition: we must have a requirement to turn $l?$ off.

$\underline{\quad TurnOff \quad}$
$\Delta DwarfSignal$
$l? : LampId$

$l? \in turn\_off$
$last\_proper\_state' = last\_proper\_state$
$turn\_off' = turn\_off \setminus \{l?\}$
$turn\_on' = turn\_on$
$last\_state' = current\_state$
$current\_state' = current\_state \setminus \{l?\}$
$desired\_proper\_state' = desired\_proper\_state$

Turning off a Lamp

- Operation on the state *DwarfSignal* with one input: $l? : LampId$.
- Precondition: we must have a requirement to turn $l?$ on.

Turning off a Lamp

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- Precondition: we must have a requirement to turn $l?$ on.

Turning off a Lamp

- Operation on the state $DwarfSignal$ with one input: $l? : LampId$.
- Precondition: we must have a requirement to turn $l?$ on.

$$
\begin{array}{l}
\textit{TurnOn} \\
\hline
\Delta DwarfSignal \\
l? : LampId \\
\hline
l? \in turn\_on \\
last\_proper\_state' = last\_proper\_state \\
turn\_off' = turn\_off \\
turn\_on' = turn\_on \setminus \{l?\} \\
last\_state' = current\_state \\
current\_state' = current\_state \cup \{l?\} \\
desired\_proper\_state' = desired\_proper\_state
\end{array}
$$

Turning off a Lamp

- Operation on the state $DwarfSignal$ with one input: $l? : LampId$.
- Precondition: we must have a requirement to turn $l?$ on.

$TurnOn$
$\Delta DwarfSignal$
$l? : LampId$

$l? \in turn\_on$
$last\_proper\_state' = last\_proper\_state$
$turn\_off' = turn\_off$
$turn\_on' = turn\_on \setminus \{l?\}$
$last\_state' = current\_state$
$current\_state' = current\_state \cup \{l?\}$
$desired\_proper\_state' = desired\_proper\_state$

Turning off a Lamp

- Operation on the state $DwarfSignal$ with one input: $l? : LampId$.
- Precondition: we must have a requirement to turn $l?$ on.

$$
\begin{array}{l}
\underline{TurnOn} \\
\Delta DwarfSignal \\
l? : LampId \\
\hline
l? \in turn\_on \\
last\_proper\_state' = last\_proper\_state \\
turn\_off' = turn\_off \\
turn\_on' = turn\_on \setminus \{l?\} \\
last\_state' = current\_state \\
current\_state' = current\_state \cup \{l?\} \\
desired\_proper\_state' = desired\_proper\_state
\end{array}
$$

Turning off a Lamp

- Operation on the state $DwarfSignal$ with one input: $l? : LampId$.
- Precondition: we must have a requirement to turn $l?$ on.

$$
\begin{array}{l}
\hline
\textit{TurnOn} \\
\Delta DwarfSignal \\
l? : LampId \\
\hline
l? \in turn\_on \\
last\_proper\_state' = last\_proper\_state \\
turn\_off' = turn\_off \\
turn\_on' = turn\_on \setminus \{l?\} \\
last\_state' = current\_state \\
current\_state' = current\_state \cup \{l?\} \\
desired\_proper\_state' = desired\_proper\_state \\
\hline
\end{array}
$$

Turning off a Lamp

- Operation on the state $DwarfSignal$ with one input: $l? : LampId$.
- Precondition: we must have a requirement to turn $l?$ on.

$$\begin{array}{l} \underline{\mathit{TurnOn}} \\ \Delta DwarfSignal \\ l? : LampId \\ \hline l? \in turn\_on \\ last\_proper\_state' = last\_proper\_state \\ turn\_off' = turn\_off \\ turn\_on' = turn\_on \setminus \{l?\} \\ last\_state' = current\_state \\ current\_state' = current\_state \cup \{l?\} \\ desired\_proper\_state' = desired\_proper\_state \end{array}$$

Turning off a Lamp

- Operation on the state $DwarfSignal$ with one input: $l? : LampId$.
- Precondition: we must have a requirement to turn $l?$ on.

$$
\begin{array}{l}
\hline
TurnOn \\
\hline
\Delta DwarfSignal \\
l? : LampId \\
\hline
l? \in turn\_on \\
last\_proper\_state' = last\_proper\_state \\
turn\_off' = turn\_off \\
turn\_on' = turn\_on \setminus \{l?\} \\
last\_state' = current\_state \\
current\_state' = current\_state \cup \{l?\} \\
desired\_proper\_state' = desired\_proper\_state \\
\hline
\end{array}
$$

Turning off a Lamp

- Operation on the state $DwarfSignal$ with one input: $l? : LampId$.
- Precondition: we must have a requirement to turn $l?$ on.

$$
\begin{array}{l}
\_\_ TurnOn _____ \\
\Delta DwarfSignal \\
l? : LampId \\
\hline
l? \in turn\_on \\
last\_proper\_state' = last\_proper\_state \\
turn\_off' = turn\_off \\
turn\_on' = turn\_on \setminus \{l?\} \\
last\_state' = current\_state \\
current\_state' = current\_state \cup \{l?\} \\
desired\_proper\_state' = desired\_proper\_state
\end{array}
$$

Turning off a Lamp

- Operation on the state $DwarfSignal$ with one input: $l? : LampId$.
- Precondition: we must have a requirement to turn $l?$ on.

$$
\begin{array}{l}
\underline{\quad TurnOn \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad} \\
\Delta DwarfSignal \\
l? : LampId \\
\underline{\quad\quad\quad\quad\quad} \\
l? \in turn\_on \\
last\_proper\_state' = last\_proper\_state \\
turn\_off' = turn\_off \\
turn\_on' = turn\_on \setminus \{l?\} \\
last\_state' = current\_state \\
current\_state' = current\_state \cup \{l?\} \\
desired\_proper\_state' = desired\_proper\_state
\end{array}
$$

Turning off a Lamp

- Operation on the state $DwarfSignal$ with one input: $l? : LampId$.
- Precondition: we must have a requirement to turn $l?$ on.

$$
\begin{array}{l}
\underline{TurnOn} \\
\Delta DwarfSignal \\
l? : LampId \\
\hline
l? \in turn\_on \\
last\_proper\_state' = last\_proper\_state \\
turn\_off' = turn\_off \\
turn\_on' = turn\_on \setminus \{l?\} \\
last\_state' = current\_state \\
current\_state' = current\_state \cup \{l?\} \\
desired\_proper\_state' = desired\_proper\_state
\end{array}
$$

Turning off a Lamp

- Operation on the state $DwarfSignal$ with one input: $l? : LampId$.
- Precondition: we must have a requirement to turn $l?$ on.

$TurnOn$
$\Delta DwarfSignal$
$l? : LampId$

$l? \in turn\_on$
$last\_proper\_state' = last\_proper\_state$
$turn\_off' = turn\_off$
$turn\_on' = turn\_on \setminus \{l?\}$
$last\_state' = current\_state$
$current\_state' = current\_state \cup \{l?\}$
$desired\_proper\_state' = desired\_proper\_state$

Turning off a Lamp

- Operation on the state $DwarfSignal$ with one input: $l? : LampId$.
- Precondition: we must have a requirement to turn $l?$ on.

$$
\begin{array}{l}
\underline{TurnOn} \\
\Delta DwarfSignal \\
l? : LampId \\
\hline
l? \in turn\_on \\
last\_proper\_state' = last\_proper\_state \\
turn\_off' = turn\_off \\
turn\_on' = turn\_on \setminus \{l?\} \\
last\_state' = current\_state \\
current\_state' = current\_state \cup \{l?\} \\
desired\_proper\_state' = desired\_proper\_state
\end{array}
$$

Overview

Use Case: *Init*

Use Case: *Init*

Use Case: *Init*

Init
DwarfSignal′

Use Case: *Init*

$\underline{\quad Init \quad\rule{10em}{0.4pt}}$

$DwarfSignal'$

$last\_proper\_state' = stop$
$turn\_off' = \emptyset$
$turn\_on' = \emptyset$
$last\_state' = stop$
$current\_state' = stop$
$desired\_proper\_state' = stop$

Use Case: $Init$

$$
\begin{array}{l}
\underline{\quad Init \quad\rule{6cm}{0pt}} \\
DwarfSignal' \\
\underline{\rule{3cm}{0pt}} \\
last\_proper\_state' = stop \\
turn\_off' = \emptyset \\
turn\_on' = \emptyset \\
last\_state' = stop \\
current\_state' = stop \\
desired\_proper\_state' = stop \\
\end{array}
$$

After $Init$, the state looks like this:

Use Case: $Init$

$$\begin{array}{|l}
\hline
\quad Init \underline{} \\
\; DwarfSignal' \\
\hline
\; last\_proper\_state' = stop \\
\; turn\_off' = \emptyset \\
\; turn\_on' = \emptyset \\
\; last\_state' = stop \\
\; current\_state' = stop \\
\; desired\_proper\_state' = stop \\
\hline
\end{array}$$

After $Init$, the state looks like this:

$$last\_proper\_state \;=\; stop$$
$$turn\_off \;=\; \emptyset$$
$$turn\_on \;=\; \emptyset$$
$$last\_state \;=\; stop$$
$$current\_state \;=\; stop$$
$$desired\_proper\_state \;=\; stop$$

Use Case: $Init \mathbin{\raise.2ex\hbox{$\fgr$}} SetNewProperState$

$$\mathbin{\raise.2ex\hbox{$\overset{\circ}{9}$}}$$

Use Case: $Init \mathbin{\text{\tiny o}_9} SetNewProperState$

$$
\left\{
\begin{array}{rcl}
last\_proper\_state' &=& stop \\
turn\_off' &=& \emptyset \\
turn\_on' &=& \emptyset \\
last\_state' &=& stop \\
current\_state' &=& stop \\
desired\_proper\_state' &=& stop
\end{array}
\right\} \mathbin{\text{\tiny o}_9}
$$

Use Case: $Init \,\raise.4ex\hbox{$\scriptscriptstyle 9$}\, SetNewProperState$

$$\left\{ \begin{aligned}
last\_proper\_state' &= stop \\
turn\_off' &= \emptyset \\
turn\_on' &= \emptyset \\
last\_state' &= stop \\
current\_state' &= stop \\
desired\_proper\_state' &= stop
\end{aligned} \right\} \,\raise.4ex\hbox{$\scriptscriptstyle 9$}$$

___SetNewProperState_____
$\Delta DwarfSignal$
$st? : ProperState$
_____
$current\_state = desired\_proper\_state$
$st? \neq current\_state$
$last\_proper\_state' = current\_state$
$turn\_off' = current\_state \setminus st?$
$turn\_on' = st? \setminus current\_state$
$last\_state' = current\_state$
$current\_state' = current\_state$
$desired\_proper\_state' = st?$

Use Case: *Init ⨟ SetNewProperState*

$$
\left\{
\begin{aligned}
last\_proper\_state' &= stop \\
turn\_off' &= \emptyset \\
turn\_on' &= \emptyset \\
last\_state' &= stop \\
current\_state' &= stop \\
desired\_proper\_state' &= stop
\end{aligned}
\right\} \; {}_9^9
$$

SetNewProperState
$\Delta DwarfSignal$
$st? : ProperState$

$current\_state = desired\_proper\_state$
$warning \neq current\_state$
$last\_proper\_state' = current\_state$
$turn\_off' = current\_state \setminus warning$
$turn\_on' = warning \setminus current\_state$
$last\_state' = current\_state$
$current\_state' = current\_state$
$desired\_proper\_state' = warning$

Use Case: *Init ⨾ SetNewProperState*

$$
\left\{
\begin{array}{rcl}
last\_proper\_state' & = & stop \\
turn\_off' & = & \emptyset \\
turn\_on' & = & \emptyset \\
last\_state' & = & stop \\
current\_state' & = & stop \\
desired\_proper\_state' & = & stop
\end{array}
\right\} \; \overset{\circ}{_\circ}
$$

___SetNewProperState_____
$\Delta DwarfSignal$
$st? : ProperState$

$stop = desired\_proper\_state$
$warning \neq stop$
$last\_proper\_state' = stop$
$turn\_off' = stop \setminus warning$
$turn\_on' = warning \setminus stop$
$last\_state' = stop$
$current\_state' = stop$
$desired\_proper\_state' = warning$

Use Case: *Init ⨾ SetNewProperState*

$$
\left\{
\begin{aligned}
last\_proper\_state' &= stop \\
turn\_off' &= \emptyset \\
turn\_on' &= \emptyset \\
last\_state' &= stop \\
current\_state' &= stop \\
desired\_proper\_state' &= stop
\end{aligned}
\right\}
$$

⨾

$$
\begin{array}{l}
\underline{SetNewProperState} \\
\Delta DwarfSignal \\
st? : ProperState \\
\hline
stop = stop \\
warning \neq stop \\
last\_proper\_state' = stop \\
turn\_off' = stop \setminus warning \\
turn\_on' = warning \setminus stop \\
last\_state' = stop \\
current\_state' = stop \\
desired\_proper\_state' = warning
\end{array}
$$

Use Case: *Init ⨟ SetNewProperState*

$$
\left\{
\begin{aligned}
last\_proper\_state' &= stop \\
turn\_off' &= \emptyset \\
turn\_on' &= \emptyset \\
last\_state' &= stop \\
current\_state' &= stop \\
desired\_proper\_state' &= stop
\end{aligned}
\right\}
\;{}_{\;9}^{9}\;
$$

___SetNewProperState_____
$\Delta DwarfSignal$
$st? : ProperState$
_____

$stop = stop$
$warning \neq stop$
$last\_proper\_state' = stop$
$turn\_off' = stop \setminus warning$
$turn\_on' = warning \setminus stop$
$last\_state' = stop$
$current\_state' = stop$
$desired\_proper\_state' = warning$

Use Case: *Init ⨾ SetNewProperState*

$$\left\{ \begin{array}{rcl} last\_proper\_state' & = & stop \\ turn\_off' & = & \emptyset \\ turn\_on' & = & \emptyset \\ last\_state' & = & stop \\ current\_state' & = & stop \\ desired\_proper\_state' & = & stop \end{array} \right\} \; \overset{\circ}{\,_\circ} \;$$

———— *SetNewProperState* ————
$\Delta DwarfSignal$
$st? : ProperState$
————
$true$
$true$
$last\_proper\_state' = stop$
$turn\_off' = stop \setminus warning$
$turn\_on' = warning \setminus stop$
$last\_state' = stop$
$current\_state' = stop$
$desired\_proper\_state' = warning$

Use Case: $Init \mathbin{\raise0.2ex\hbox{$\scriptstyle\circ$}\raise-0.2ex\hbox{$\scriptstyle\circ$}} SetNewProperState$

$$
\left\{
\begin{aligned}
last\_proper\_state' &= stop \\
turn\_off' &= \emptyset \\
turn\_on' &= \emptyset \\
last\_state' &= stop \\
current\_state' &= stop \\
desired\_proper\_state' &= stop
\end{aligned}
\right\} \mathbin{\raise0.2ex\hbox{$\scriptstyle\circ$}\raise-0.2ex\hbox{$\scriptstyle\circ$}}
$$

$\underline{SetNewProperState \underline{\qquad\qquad\qquad}}$
$\Delta DwarfSignal$
$st? : ProperState$

$last\_proper\_state' = stop$
$turn\_off' = stop \setminus warning$
$turn\_on' = warning \setminus stop$
$last\_state' = stop$
$current\_state' = stop$
$desired\_proper\_state' = warning$

Use Case: *Init ⨾ SetNewProperState*

$$
\left\{
\begin{aligned}
last\_proper\_state' &= stop \\
turn\_off' &= \emptyset \\
turn\_on' &= \emptyset \\
last\_state' &= stop \\
current\_state' &= stop \\
desired\_proper\_state' &= stop
\end{aligned}
\right\} \;{}_{9}^{9}
$$

$$
\begin{array}{l}
\underline{\textit{SetNewProperState}} \underline{\hspace{4cm}} \\
\Delta DwarfSignal \\
st? : ProperState \\
\hline
last\_proper\_state' = stop \\
turn\_off' = \{L1, L2\} \setminus \{L1, L3\} \\
turn\_on' = \{L1, L3\} \setminus \{L1, L2\} \\
last\_state' = stop \\
current\_state' = stop \\
desired\_proper\_state' = warning
\end{array}
$$

Use Case: *Init ⨟ SetNewProperState*

$$
\left\{
\begin{array}{rcl}
last\_proper\_state' &=& stop \\
turn\_off' &=& \emptyset \\
turn\_on' &=& \emptyset \\
last\_state' &=& stop \\
current\_state' &=& stop \\
desired\_proper\_state' &=& stop
\end{array}
\right\} \;_{\overset{\circ}{\circ}}
$$

SetNewProperState _____
$\Delta DwarfSignal$
$st? : ProperState$

$last\_proper\_state' = stop$
$turn\_off' = \{L2\}$
$turn\_on' = \{L3\}$
$last\_state' = stop$
$current\_state' = stop$
$desired\_proper\_state' = warning$

Use Case: Installation then Set to Warning

1. **Init**:

$$last\_proper\_state = stop$$
$$turn\_off = \emptyset$$
$$turn\_on = \emptyset$$
$$last\_state = stop$$
$$current\_state = stop$$
$$desired\_proper\_state = stop$$

2. **SetNewProperState warning**:

$$last\_proper\_state = stop$$
$$turn\_off = stop \setminus warning$$
$$= \{L1, L2\} \setminus \{L1, L3\}$$
$$= \{L2\}$$
$$turn\_on = warning \setminus stop$$
$$= \{L1, L3\} \setminus \{L1, L2\}$$
$$= \{L3\}$$
$$last\_state = stop$$
$$current\_state = stop$$
$$desired\_proper\_state = warning$$

3. **TurnOff L2**:

$$last\_proper\_state = stop$$
$$turn\_off = \{L2\} \setminus \{L2\}$$
$$= \emptyset$$
$$turn\_on = \{L3\}$$
$$last\_state = stop$$
$$current\_state = stop \setminus \{L2\}$$
$$= \{L1, L2\} \setminus \{L2\}$$
$$= \{L1\}$$
$$desired\_proper\_state = warning$$

4. **TurnOn L3**:

$$last\_proper\_state = stop$$
$$turn\_off = \emptyset$$
$$turn\_on = \{L3\} \setminus \{L3\}$$
$$= \emptyset$$
$$last\_state = \{L1\}$$
$$current\_state = \{L1\} \cup \{L3\}$$
$$= \{L1, L3\}$$
$$= warning$$
$$desired\_proper\_state = warning$$

Use Case: Installation then Set to Warning

1. **Init**:

$$
\begin{aligned}
last\_proper\_state &= stop \\
turn\_off &= \emptyset \\
turn\_on &= \emptyset \\
last\_state &= stop \\
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\end{aligned}
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$$
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last\_proper\_state &= stop \\
turn\_off &= stop \setminus warning \\
&= \{L1, L2\} \setminus \{L1, L3\} \\
&= \{L2\} \\
turn\_on &= warning \setminus stop \\
&= \{L1, L3\} \setminus \{L1, L2\} \\
&= \{L3\} \\
last\_state &= stop \\
current\_state &= stop \\
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\end{aligned}
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turn\_on &= \{L3\} \\
last\_state &= stop \\
current\_state &= stop \setminus \{L2\} \\
&= \{L1, L2\} \setminus \{L2\} \\
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current\_state &= \{L1\} \cup \{L3\} \\
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Conclusion

- Described a real-world railway signalling device: the dwarf signal.

- Specified the behaviour in the Z notation, starting with the state.

- Formalised the safety requirements.

- Specified the user interface: one operation plus two daemon operations.

- Described a use case.

- Final lecture: how to automate the specification animation.

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