Z Machines

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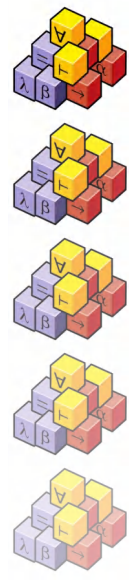
Overview

1. Modelling Z Specifications with Z Machines
2. Types, Stores, and State Variables
3. Operations and Machines
4. Animation and Verification
5. Managing Requirements



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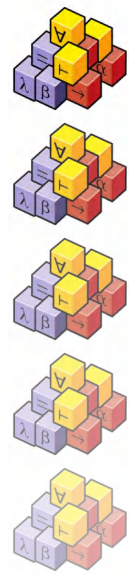

Motivation

- The Z notation is a rich specification language:

- This flexibility can however make it difficult to automate in practice.
- Z Machines: restricted subset of Z in Isabelle/HOL (like Event-B).
- A design pattern for the more general Z language.
- Closer to an implementation and requires some design decisions.
- Careful handling of types vs. set dichotomy, which is less visible in Z.
- Includes support for (1) proof obligation generation and (2) animation.

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- The Z notation is a rich specification language:

IncubatorMonitor

$temp : \mathbb{Z}$

$MIN \leq temp \leq MAX$

Increment

$\Delta IncubatorMonitor$

$temp < MAX$

$temp' = temp + 1$

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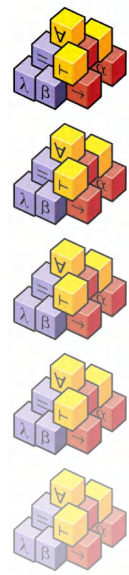

Example: Dwarf Signal Types (Z)

$$LampId ::= L1 \mid L2 \mid L3$$

$$dark, stop, warning, drive : \mathbb{F}\, LampId$$

$$dark = \emptyset$$
$$stop = \{L1, L2\}$$
$$warning = \{L1, L3\}$$
$$drive = \{L2, L3\}$$

$$ProperState == \{dark, stop, warning, drive\}$$

$$Signal == \mathbb{F}\, LampId$$

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enumtype LampId = L1 | L2 | L3

type_synonym Signal = "LampId set"

enumtype ProperState = dark | stop | warning | drive

definition "ProperState = {dark, stop, warning, drive}"

fun signalLamps :: "ProperState ⇒ LampId set" where
  "signalLamps dark = {}" |
  "signalLamps stop = {L1, L2}" |
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Stores

- A store is a set of variable declarations to be present in the state.
- Corresponds to a state schema in Z (no dashed variables).
- Introduces several state components (`x`, `y`, `z`) and invariants.
- Invariants can be optionally named and are collected in `state_inv`.

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State Schema and ZStore

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Dwarf

$$last\_proper\_state : ProperState$$
$$turn\_off, turn\_on : \mathbb{F}\ LampId$$
$$last\_state, current\_state : Signal$$
$$desired\_proper\_state : ProperState$$

$$(current\_state \setminus turn\_off) \cup turn\_on = desired\_proper\_state$$
$$turn\_off \cap turn\_on = \emptyset$$

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  last_proper_state :: "ProperState"
  turn_off :: "LampId set"
  turn_on  :: "LampId set"
  last_state :: "Signal"
  current_state :: "Signal"
  desired_proper_state :: "ProperState"
where
  "(current_state - turn_off) ∪ turn_on = signalLamps desired_proper_state"
  "turn_on ∩ turn_off = {}"
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Dwarf Signal Requirements Schema

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NeverShowAll

Dwarf

$$current\_state \neq \{L1, L2, L3\}$$

Dwarf Signal Requirements Schema

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$current\_state \neq \{L1, L2, L3\}$

DwarfSignal

NeverShowAll

MaxOneLampChange

ForbidStopToDrive

DarkOnlyToStop

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Dwarf Signal Requirements Schema

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We deal with these separately in Z Machines.

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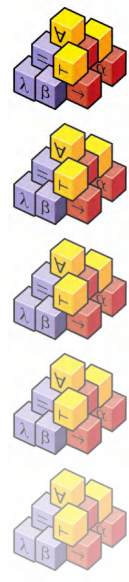

Operations

- zoperation corresponds to a Z operation schema, but restricted.
- Inputs ($a?$) and outputs ($b!$) are modelled by parameters.
- Parameters come from sets, not types. Can depend on the state.
- Preconditions can depend only on the undashed before state.
- Assignments update variables. Unmentioned variables unchanged.
- State invariants are not imposed; we need to prove they are preserved.
- Much closer to an implementation of the model.

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Operation to Set New Proper State

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SetNewProperState

$\Delta DwarfSignal$

$st? : ProperState$

$current\_state = desired\_proper\_state$

$st? \neq current\_state$

$last\_proper\_state' = current\_state$

$turn\_off' = current\_state \setminus st?$

$turn\_on' = st? \setminus current\_state$

$last\_state' = current\_state$

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zoperation SetNewProperState =
  over Dwarf
  params st∈"ProperState -
              {desired_proper_state}"
  pre "current_state =
         signalLamps desired_proper_state"
  update "[
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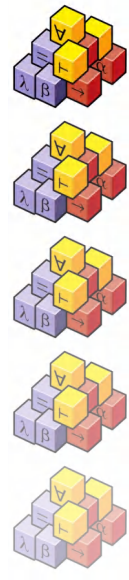

Machines and Animation

- Often implicit in the Z notation; the set of all operations.
- Generates an interaction tree semantics, for animation.
- Animate using `animate` command. Useful for design-space exploration.
- Sets initial state, checks which operations + parameters are enabled.
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- Requirements specified as machine invariants (cf. $DwarfSignal$).
- Machine verification requires we demonstrate:
  1. The initial state assignment establishes the invariants.
  2. Each operation preserves the invariants.
- Specified as Hoare conjectures: $\{state\_inv\}\ Op1(x, y, z)\ \{state\_inv\}$.
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lemma Init_inv: "Init establishes state_inv"

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  2. Each operation preserves the invariants.
- Specified as Hoare conjectures: $\{state\_inv\}\ Op1(x, y, z)\ \{state\_inv\}$.
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- Requirements specified as machine invariants (cf. *DwarfSignal*).

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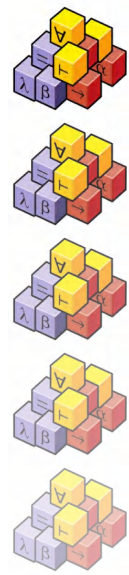
Overview



Managing Requirements

- Requirements characterised by named assertions using `zexpr`.
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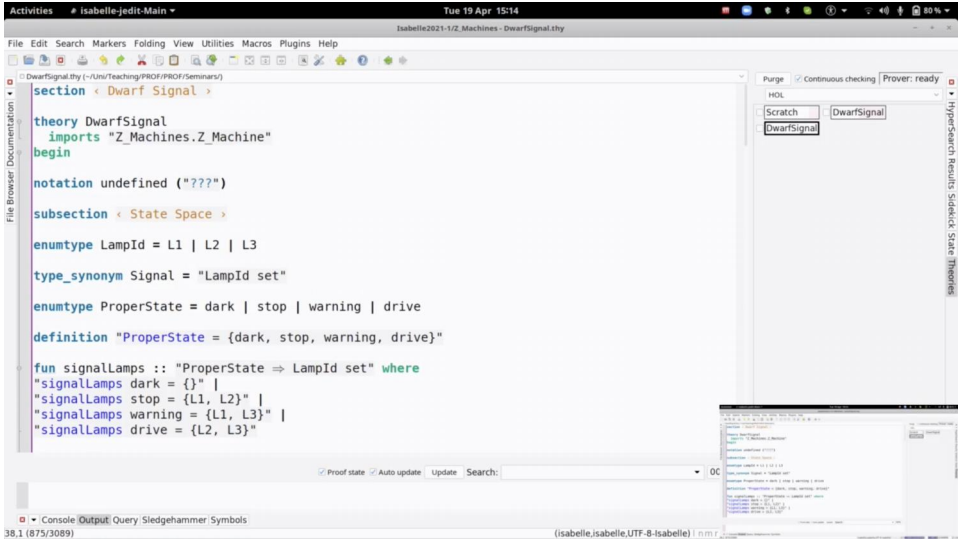

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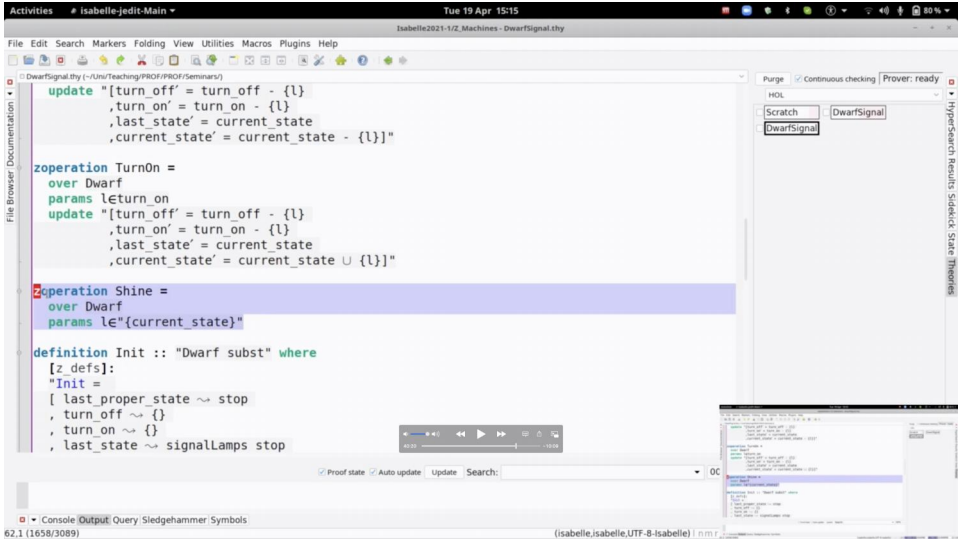

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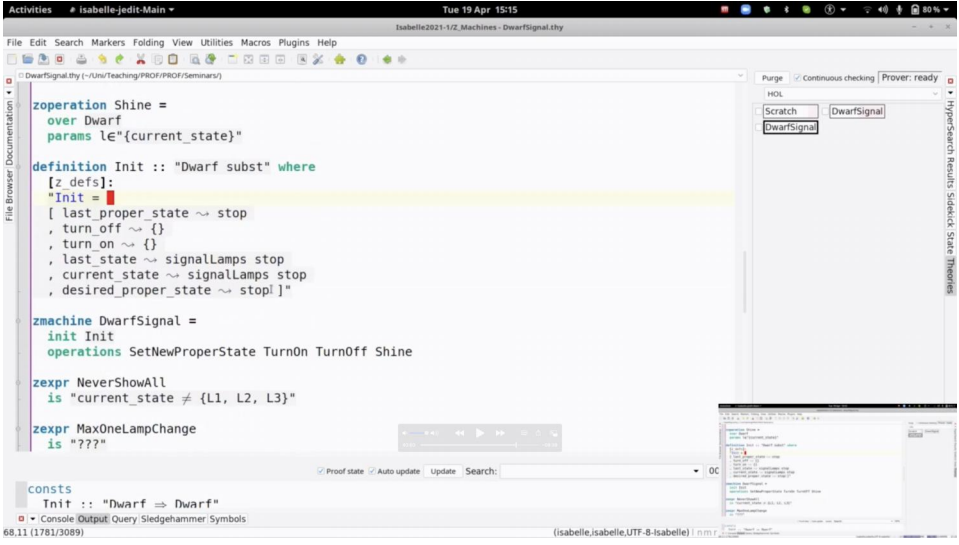

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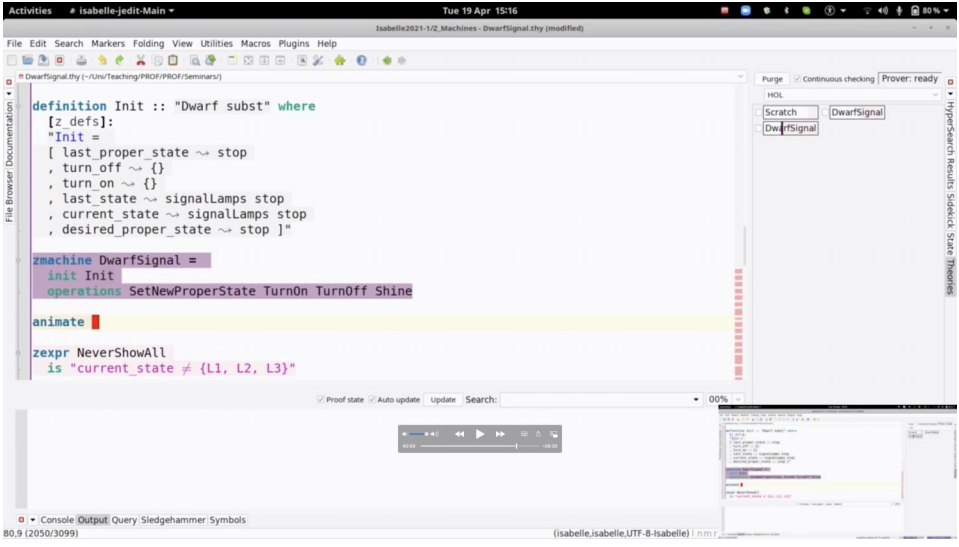

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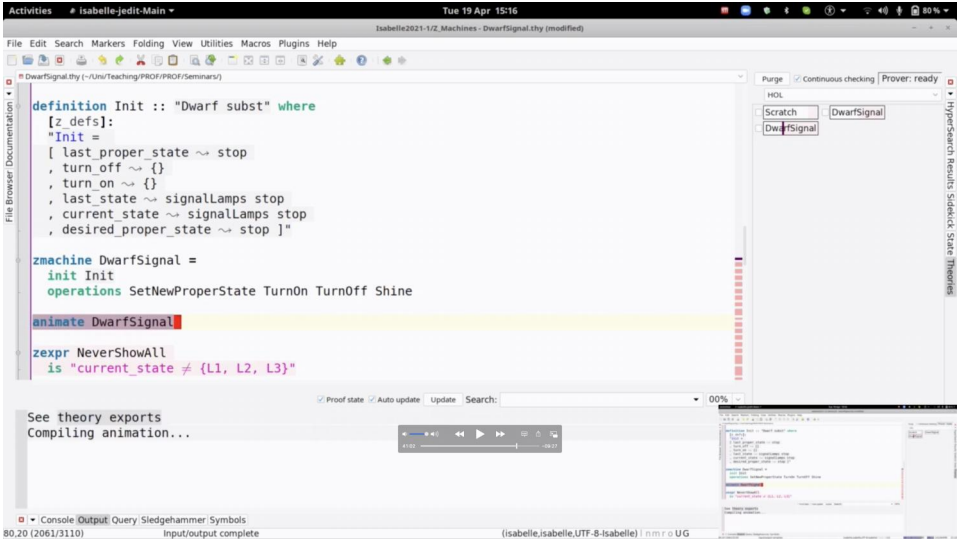

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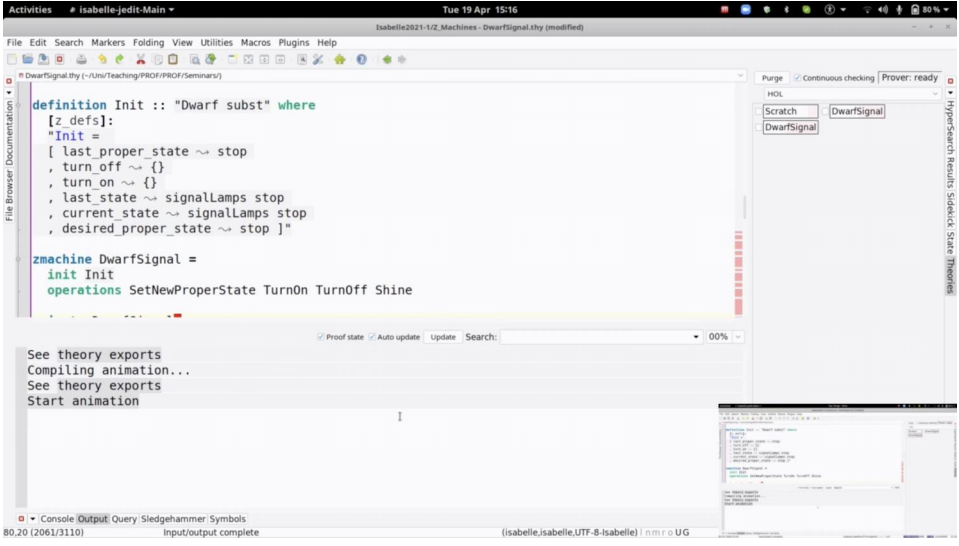

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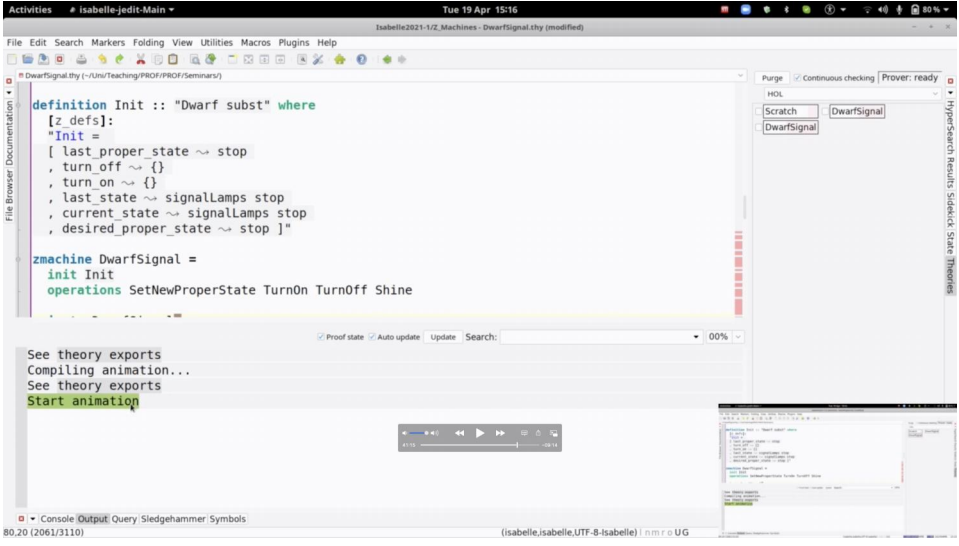

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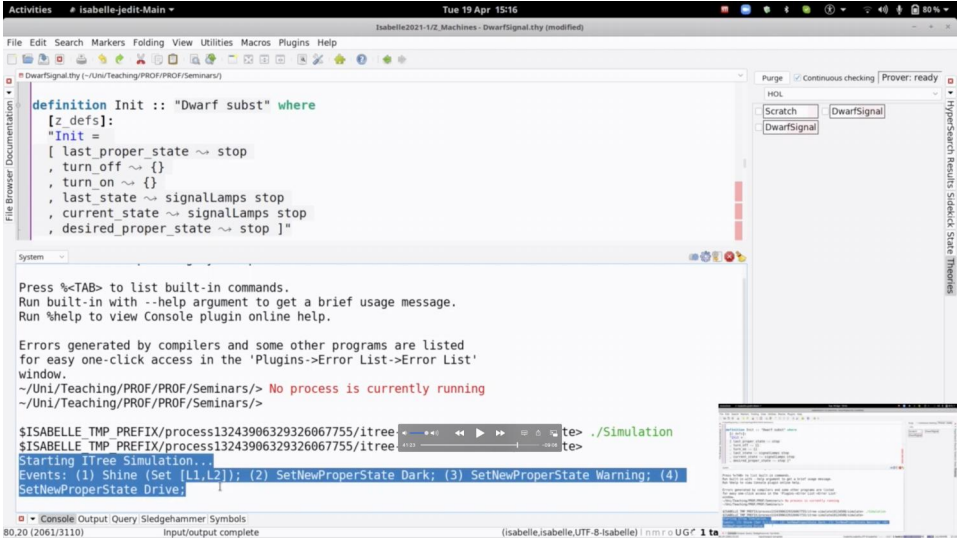

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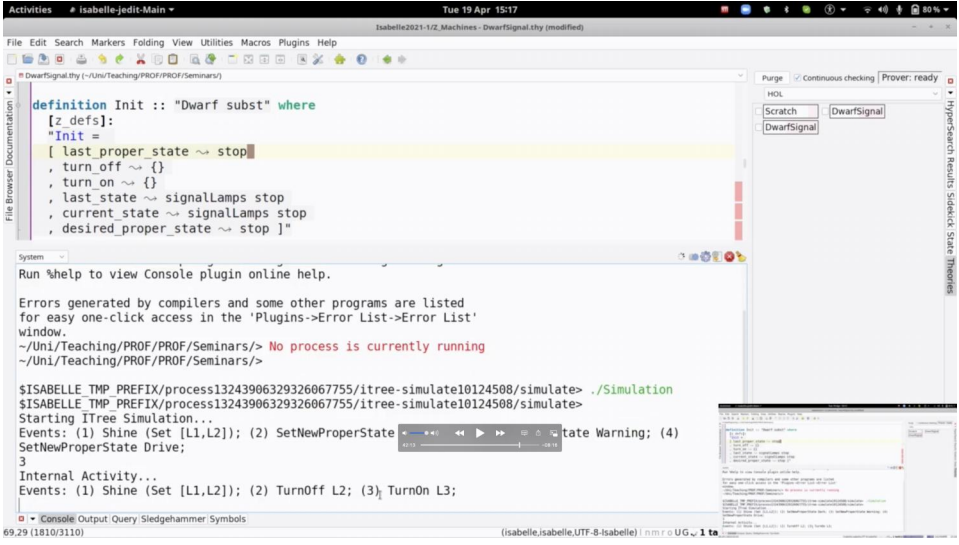

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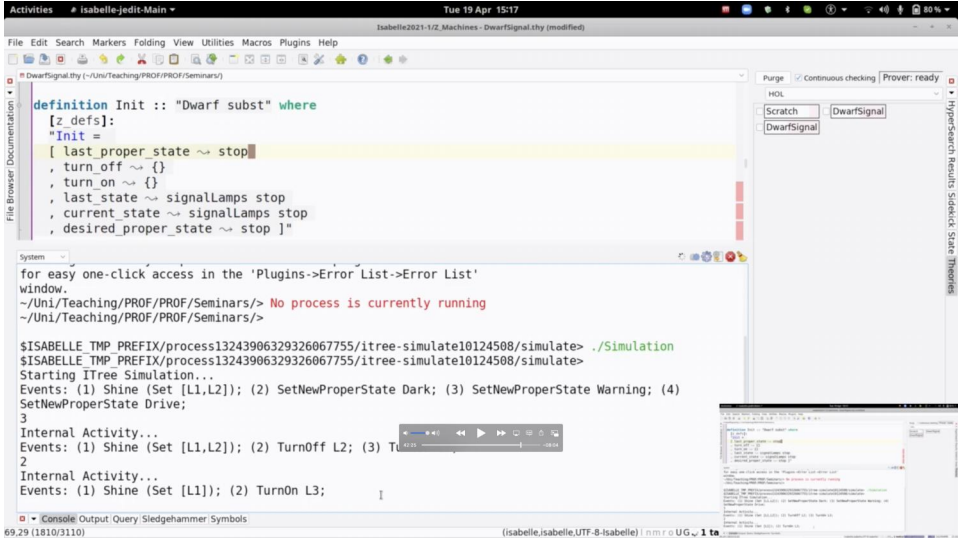

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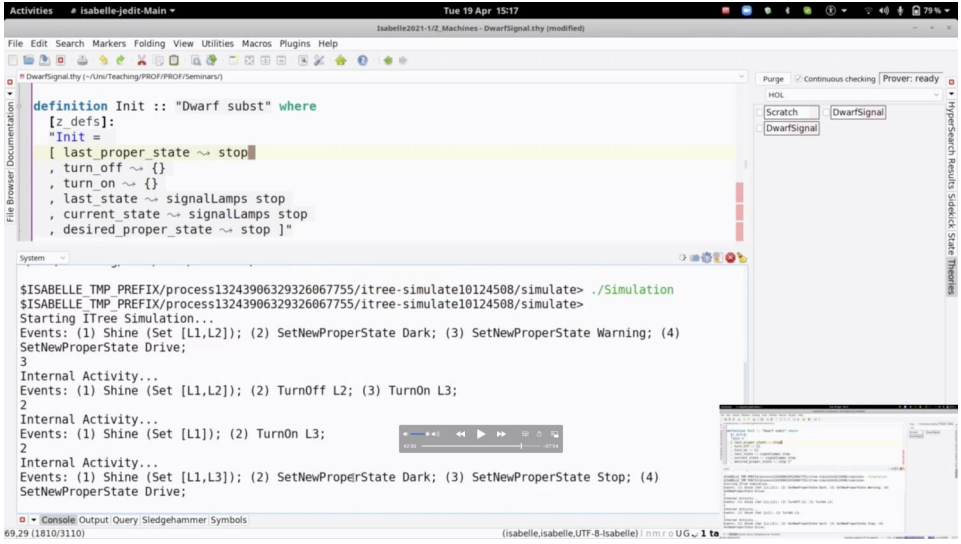

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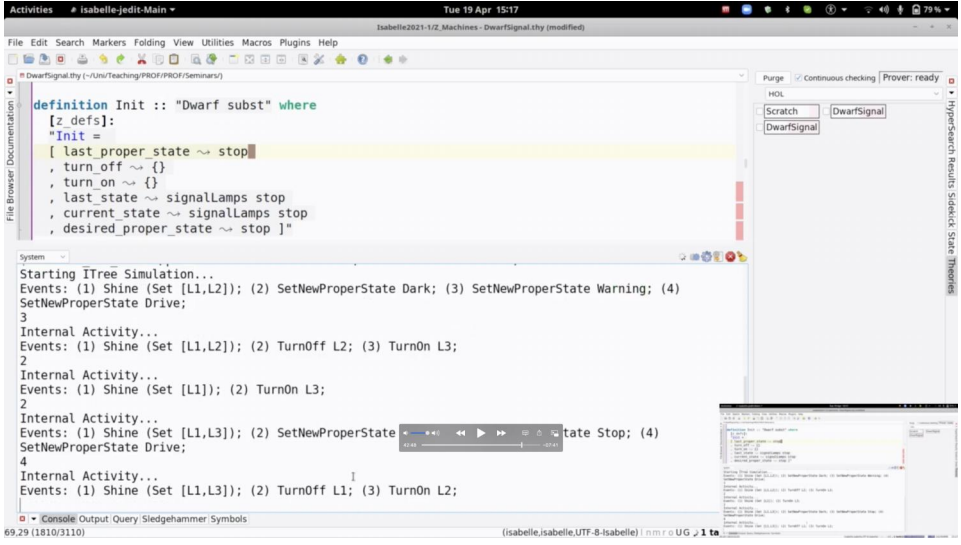

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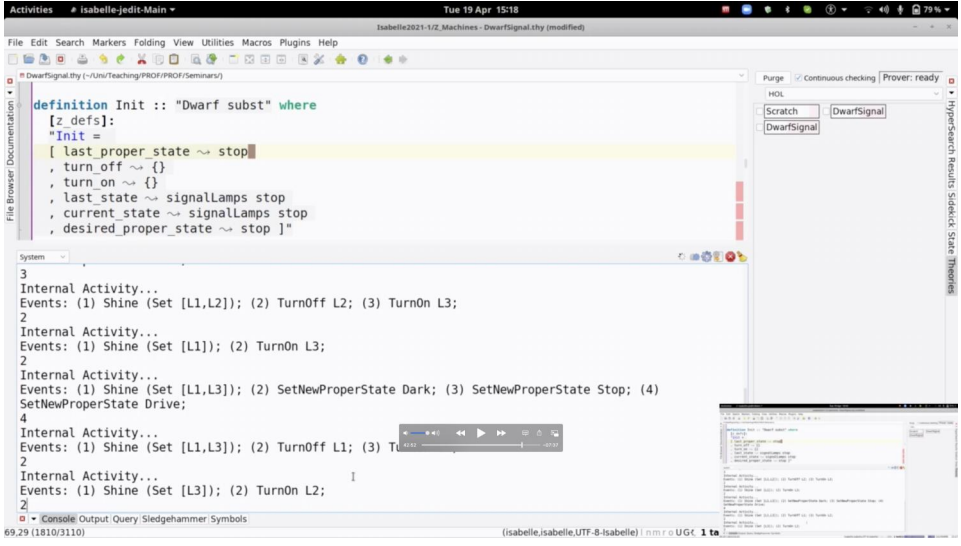

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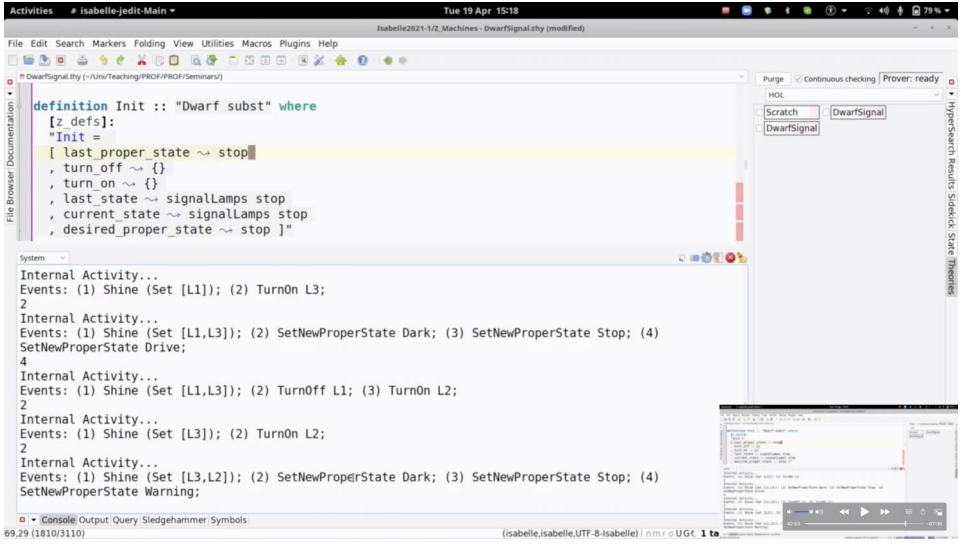

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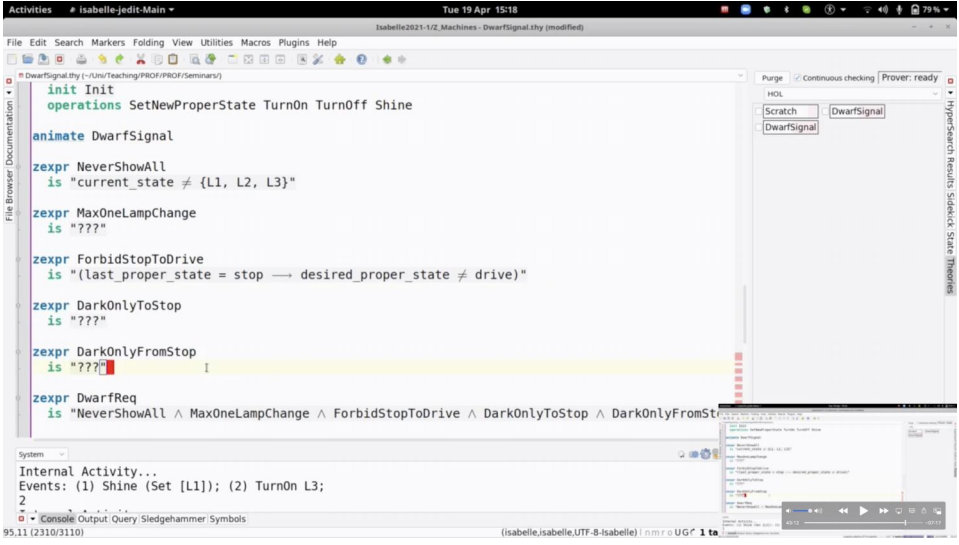

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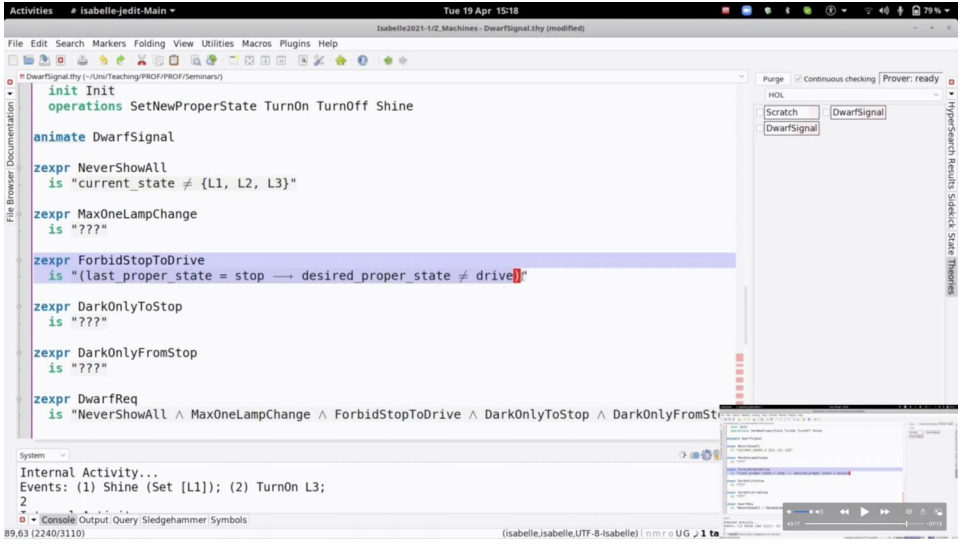

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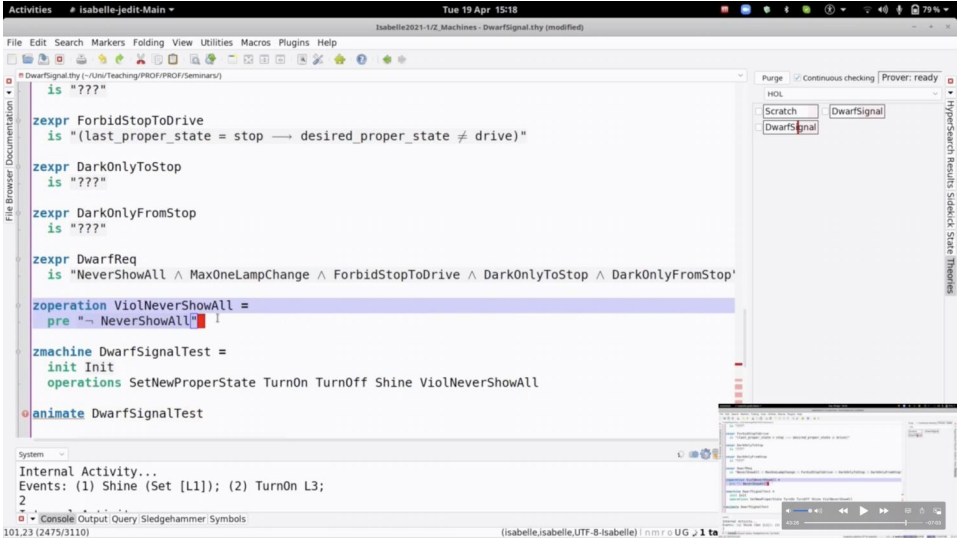

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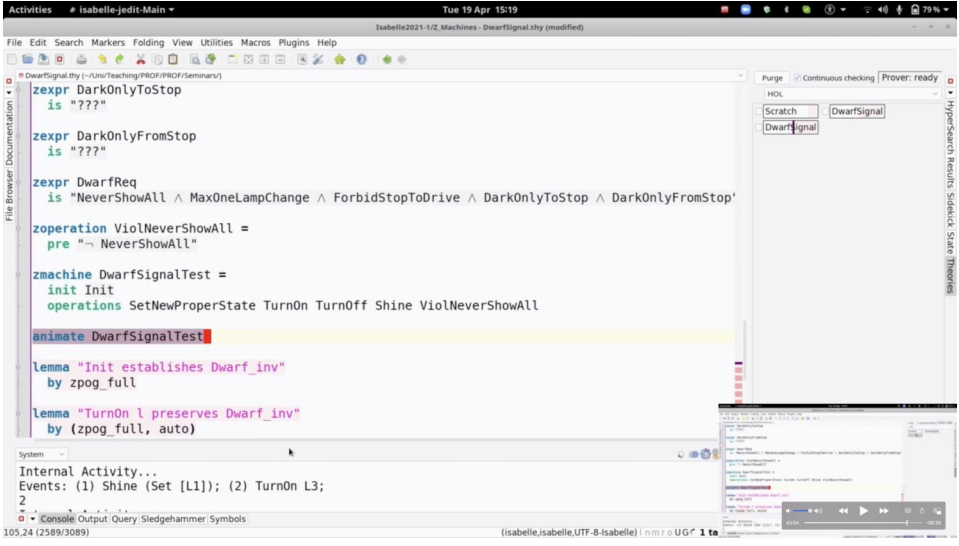

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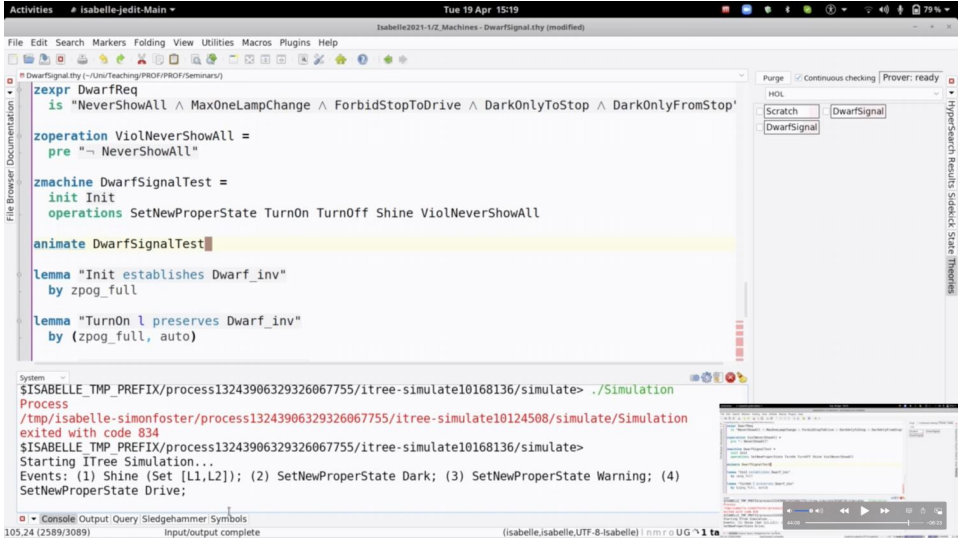

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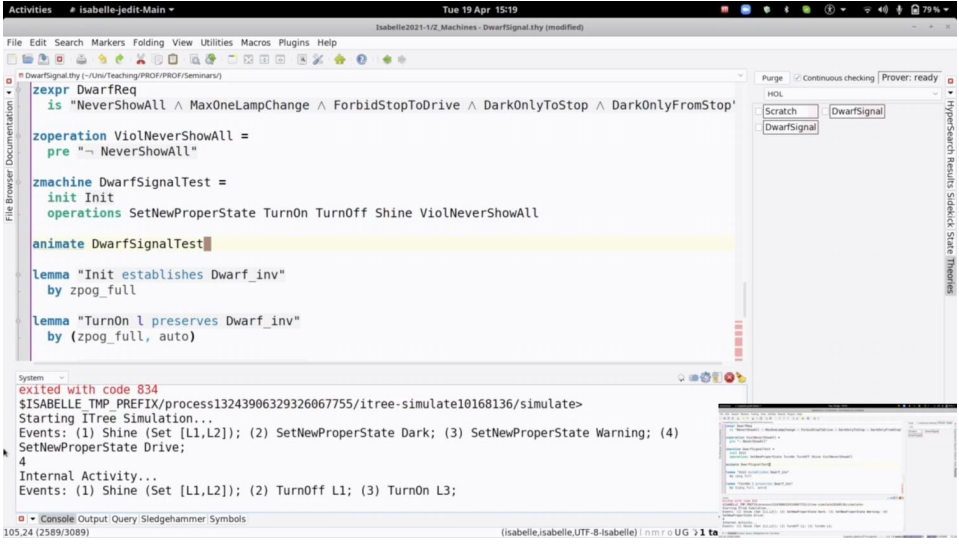

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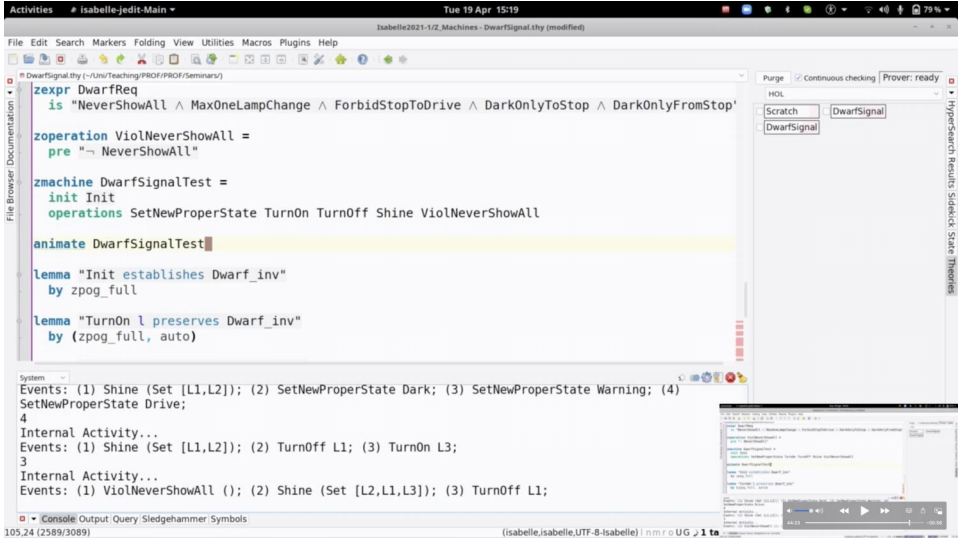

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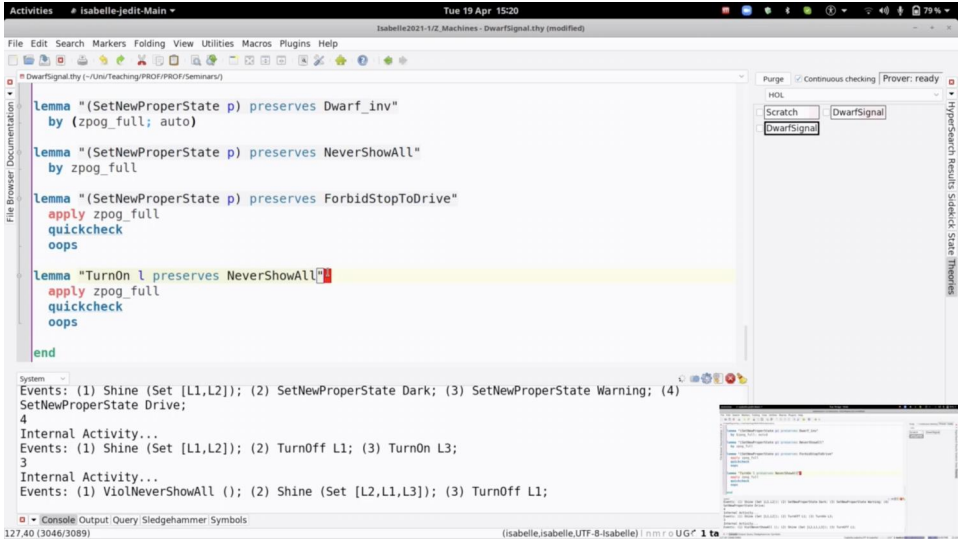

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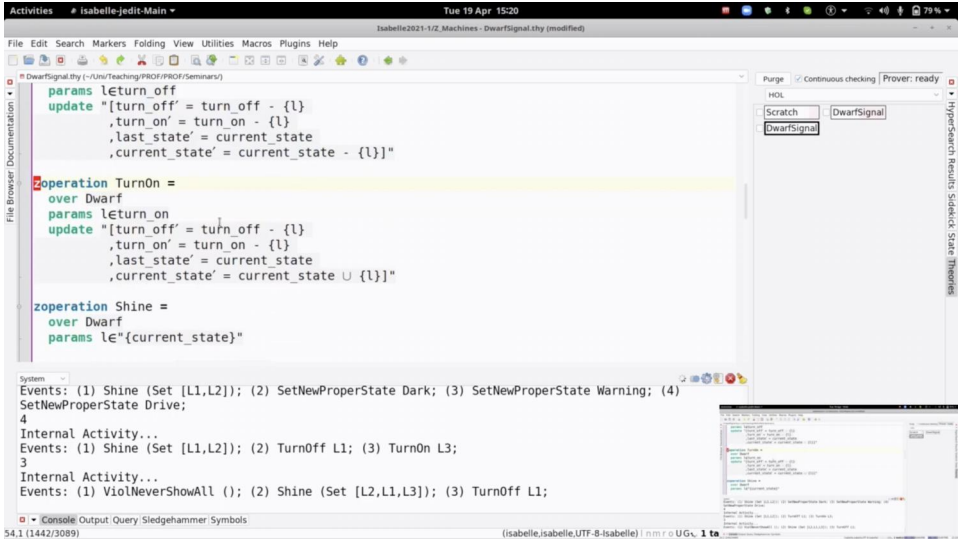

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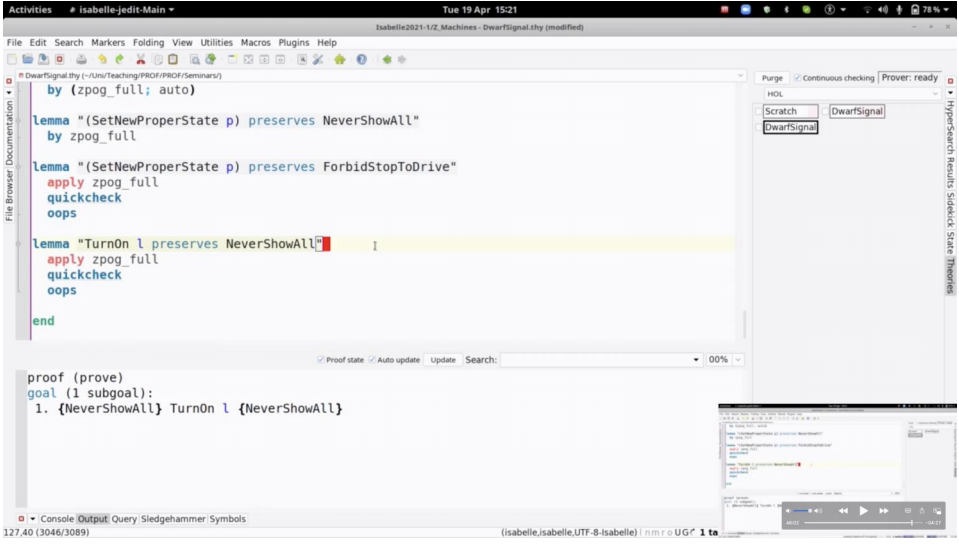

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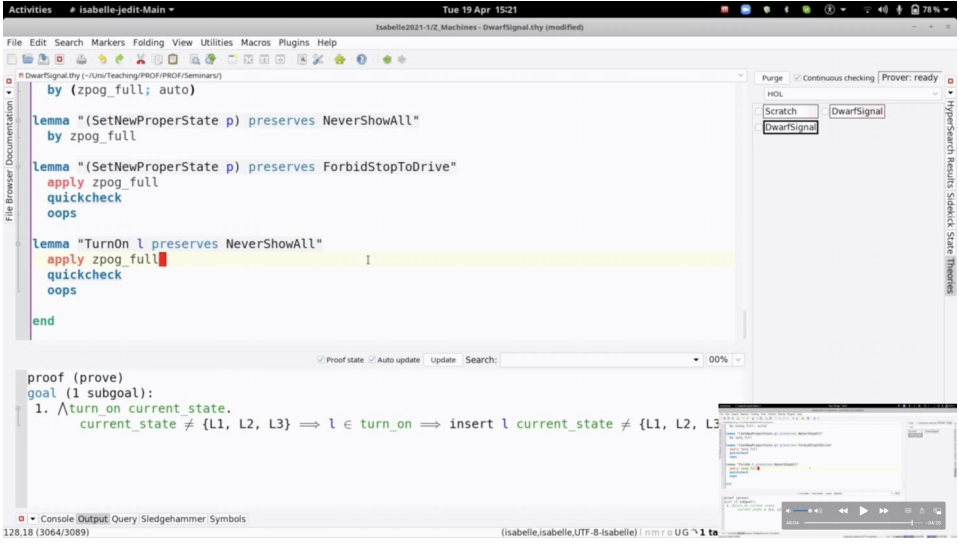

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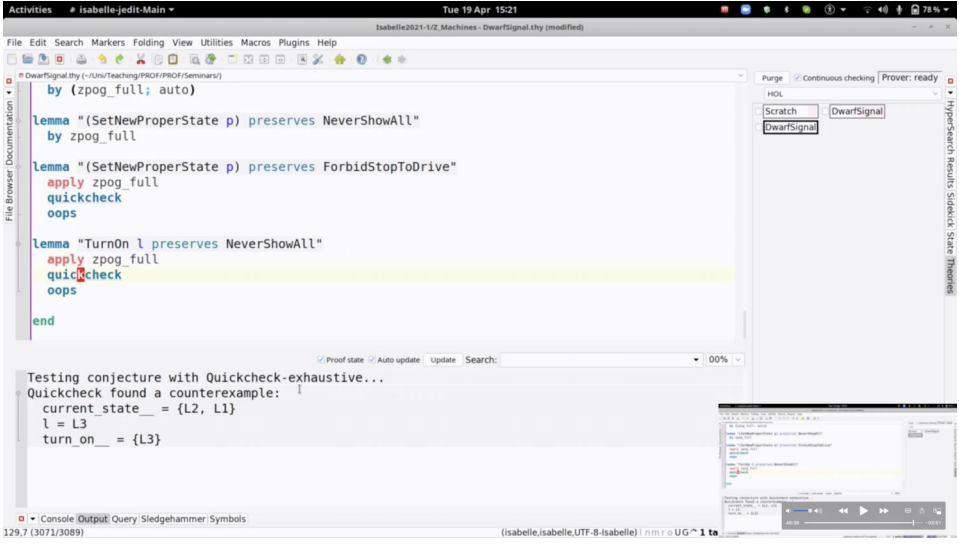

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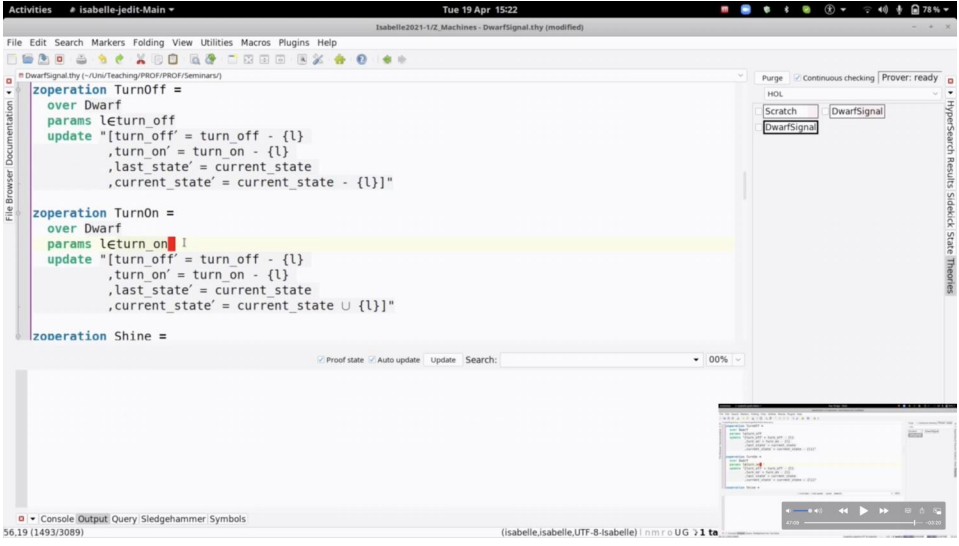

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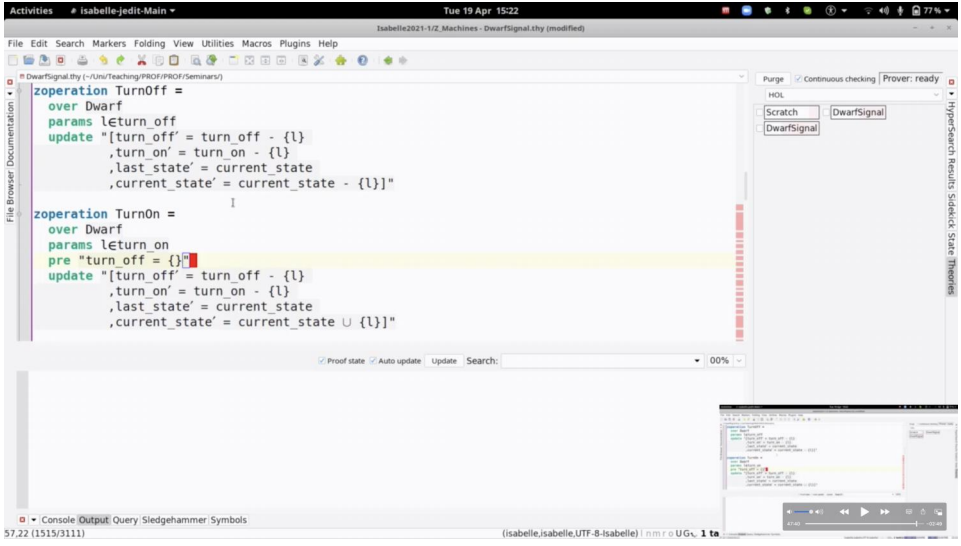

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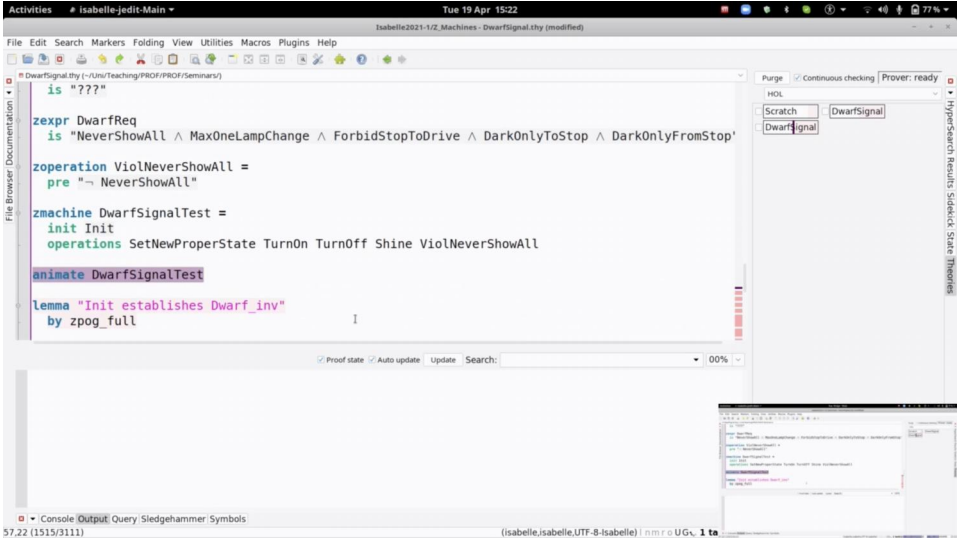

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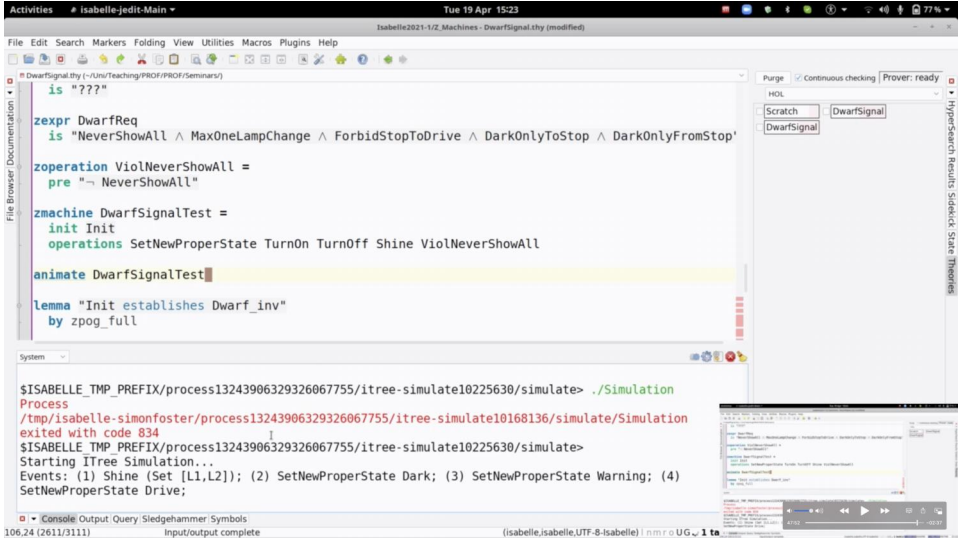

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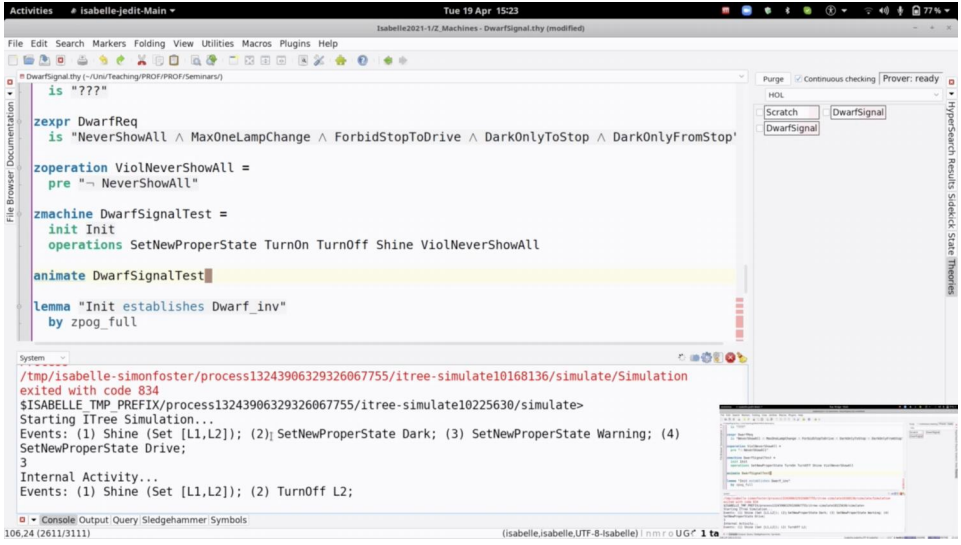

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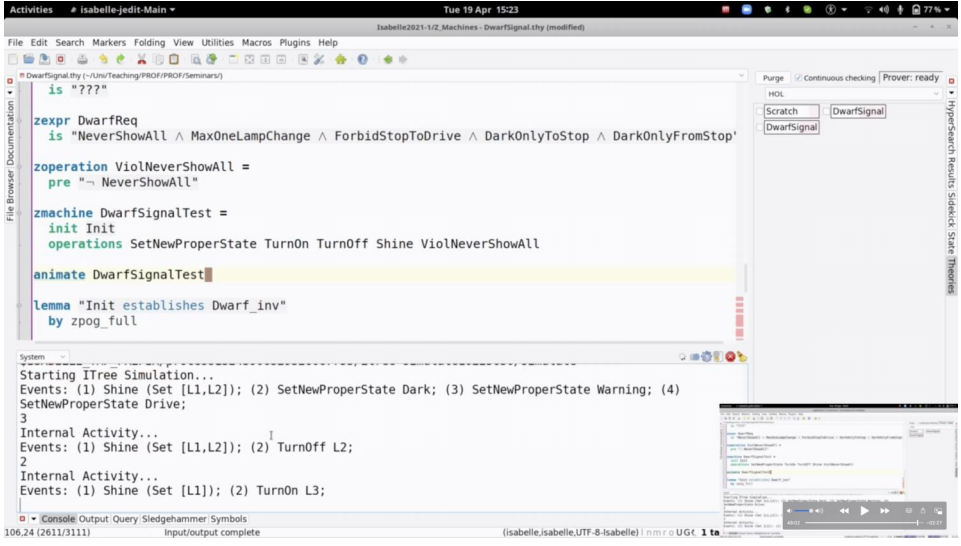

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- Operations and machines.
- Animation and verification.
- Managing requirements.

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